

STRUCTURE FUNCTION F_L AT FIXED W IN THE k_T -FACTORIZATION APPROACH

A.V. Kotikov¹, A.V. Lipatov² and N.P. Zotov^{2†}

(1) *BLThPh, Joint Institute for Nuclear Research, 141980 Dubna, Russia*
 (2) *Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University
 119992 Moscow, Russia*
 † *E-mail: zotov@theory.sinp.msu*

Abstract

The results for structure function F_L , obtained in the k_T -factorization and collinear approaches, are compared with recent H1 experimental data at fixed W values.

1 Introduction

The longitudinal structure function (SF) $F_L(x, Q^2)$ is a very sensitive QCD characteristic and is directly connected to the gluon content of the proton. It is equal to zero in the parton model with spin-1/2 partons and has got nonzero values in the framework of perturbative Quantum Chromodynamics. The perturbative QCD, however, leads to a quite controversial results. At the leading order (LO) approximation F_L amounts to about 10 ÷ 20% of the corresponding F_2 values at large Q^2 range and, thus, it has got quite large contributions at low x range. The next-to-leading order (NLO) corrections to the longitudinal coefficient function are large and negative at small x [1]-[3] and can lead to negative F_L values at low x and low Q^2 values (see [3, 4]). Negative F_L values demonstrate a limitations of the applicability of perturbation theory and the necessity of a resummation procedure, that leads to coupling constant scale higher than Q^2 (see [3], [5]-[7]).

The experimental extraction of F_L data requires a rather cumbersome procedure, especially at small values of x . Recently, however, there have been presented new precise preliminary H1 data [8] on the longitudinal SF F_L , which have probed the small- x region $10^{-5} \leq x \leq 10^{-2}$.

In Ref. [9] the standard perturbative QCD formulas and also the so called k_T -factorization approach [10] based on Balitsky-Fadin-Kuraev-Lipatov (BFKL) dynamics [11] are used for the analysis of the above data. Here we present the main results of our analysis.

In the framework of the k_T -factorization approach a study of the longitudinal SF F_L has been done firstly in Ref. [12]. We follow a more phenomenological approach [13](see also [14, 15]), where we analyzed F_L data in a broader range at small x , using the different parameterizations of the unintegrated gluon distribution function $\Phi_g(x, k_\perp^2)$ (see Ref. [16]).

2 Theoretical framework

The unintegrated gluon distribution $\Phi_g(x, k_\perp^2)$ (f_g is the (integrated) gluon distribution in the proton multiplied by x and k_\perp is the transverse part of the gluon 4-momentum k^μ)

$$f_g(x, Q^2) = \int_x^{Q^2} dk_\perp^2 \Phi_g(x, k_\perp^2) \quad (\text{hereafter } k^2 = -k_\perp^2) \quad (1)$$

is the basic dynamical quantity in the k_T -factorization approach. It satisfies the BFKL equation [11].

Then, in the k_T -factorization the SF $F_{2,L}(x, Q^2)$ are driven at small x primarily by gluons and are related in the following way to $\Phi_g(x, k_\perp^2)$:

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dz}{z} \int^{Q^2} dk_\perp^2 \sum_{i=u,d,s,c} e_i^2 \cdot \hat{C}_{2,L}^g(x/z, Q^2, m_i^2, k_\perp^2) \Phi_g(z, k_\perp^2), \quad (2)$$

where e_i^2 are charge squares of active quarks.

The functions $\hat{C}_{2,L}^g(x, Q^2, m_i^2, k_\perp^2)$ can be regarded as SF of the off-shell gluons with virtuality k_\perp^2 (hereafter we call them *hard structure functions* by analogy with similar relations between cross-sections and hard cross-sections). They are described by the sum of the quark box (and crossed box) diagram contribution to the photon-gluon interaction (see, for example, Fig. 1 in [13]).

Notice that the k_\perp^2 -integral in Eqs. (1) and (2) can be divergent at lower limit, at least for some parameterizations of $\Phi_g(x, k_\perp^2)$. To overcome the problem we change the low Q^2 asymptotics of the QCD coupling constant within hard structure functions. We applied the “freezing” procedure [17], which contains the shift $Q^2 \rightarrow Q^2 + M^2$, where M is an additional scale, which strongly modifies the infrared α_s properties. For massless produced quarks, ρ -meson mass m_ρ is usually taken as the M value, i.e. $M = m_\rho$. In the case of massive quarks with mass m_i , the $M = 2m_i$ value is usually used. For the unintegrated gluon distribution $\Phi(x, k_\perp^2, Q_0^2)$ we use the so-called Blumlein’s parametrization (JB) [18]. Note that there are also several other popular parameterizations, which give quite similar results excepting, perhaps, the contributions from the small k_\perp^2 -range: $k_\perp^2 \leq 1 \text{ GeV}^2$ (see Ref. [16]).

The JB parametrization depends strongly on the Pomeron intercept value. In different models the Pomeron intercept has different values. So, in our calculations we apply the H1 parameterization [19], which are in good agreement with perturbative QCD.

We calculate the SF F_L as the sum of two types of contributions - the charm quark one F_L^c and the light quark one F_L^l :

$$F_L = F_L^l + F_L^c. \quad (3)$$

For the F_L^l part we use the massless limit of hard SF (see [13]). We always use $f = 4$ in our fits, because our results depend very weakly on the exact f value.

3 Numerical results

In Fig. 1 we show the SF F_L with “frozen” coupling constant as a function of Q^2 for fixed W in comparison with H1 experimental data [8]. The k_T -factorization results lie

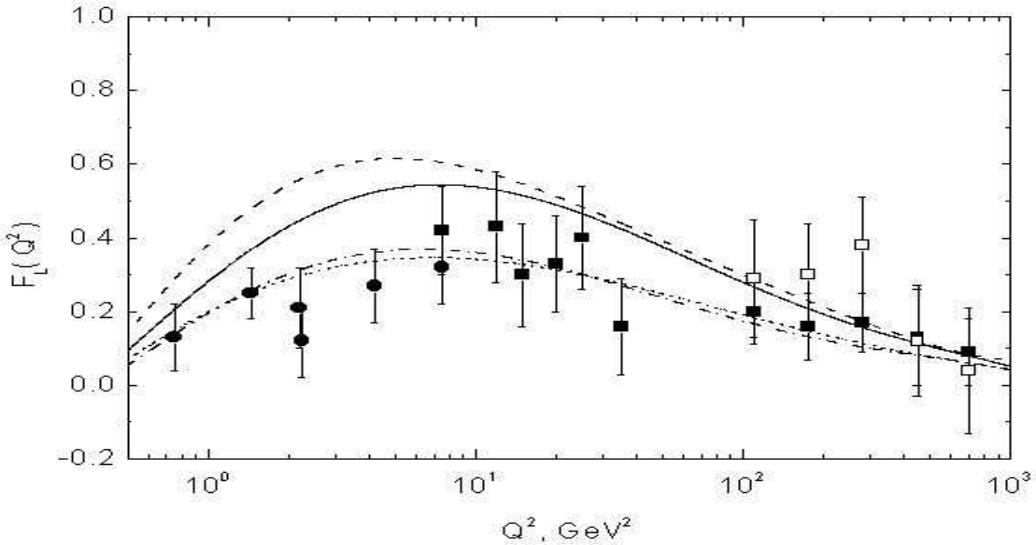


Figure 1: Q^2 dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The experimental points are from [8]. Solid curve is the result of the k_T -factorization approach, dashed, dash-dotted and dotted curves - the results of the collinear LO, NLO and LO with $\mu^2 = 127Q^2$ calculations, respectively.

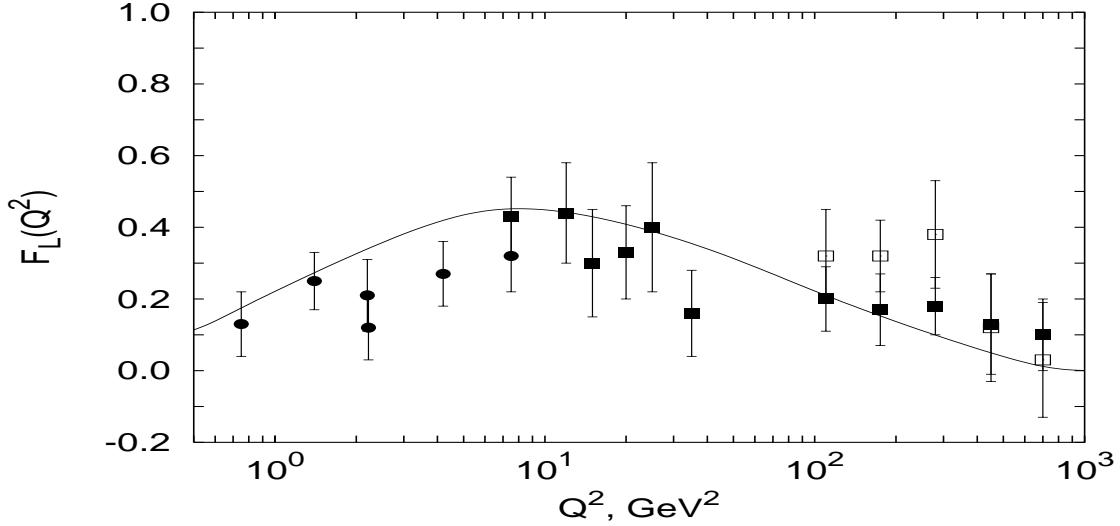


Figure 2: Q^2 dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. Solid curve is the result of the k_T -factorization approach with the GLLM unintegrated gluon distribution from [23].

between the collinear ones, that demonstrates clearly the particular resummation of high-order collinear contributions at small x values in the k_T -factorization approach. We also see excellent agreement between the experimental data and collinear approach with GRV parton densities [20] at NLO approximation (the corresponding coefficient functions were taken from the papers [1]). The NLO corrections are large and negative and decrease the F_L value by an approximate factor of 2 at $Q^2 < 10$ GeV 2 . Our k_T -factorization results

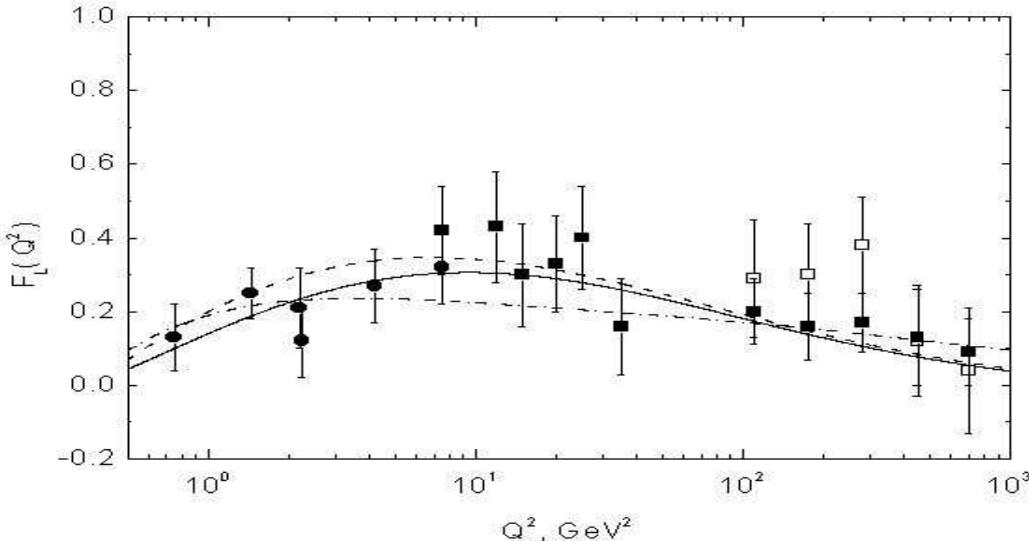


Figure 3: Q^2 dependence of $F_L(x, Q^2)$ (at fixed $W = 276$ GeV). The experimental points are as in Fig. 1. Solid curve is the result of the k_T -factorization approach at $\mu^2 = 127Q^2$, dashed curve - the collinear LO calculations at $\mu^2 = 127Q^2$, dash-dotted curve - from the R_{world} -parametrization.

are in good agreement with the data for large and small parts of the Q^2 range. We have, however, some disagreement between the data and theoretical predictions at $Q^2 \sim 3$ GeV 2 . The disagreement exists in both cases: for collinear QCD approach at the LO approximation and for k_T -factorization. It is possible to assume, that the disagreement comes from two reasons: additional higher-twist contributions, which are important at low Q^2 values¹, or/and NLO QCD corrections.

It was shown that the saturation (non-linear QCD) approaches contain information of all orders in $1/Q^2$, they resum higher-twist contributions [21]. The analysis of the behaviour of the longitudinal structure function $F_L(x, Q^2)$ in the saturation models was done in Ref. [22]². In Fig. 2 we demonstrate our k_T -factorization description of $F_L(Q^2)$ at fixed W with the unintegrated gluon distribution proposed in Ref. [23] which takes into account non-linear (saturation) effects.

Concerning the NLO corrections in the k_T -factorization approach a rough estimation of that can be done in the following way. Consider firstly the BFKL approach. A popular resummation of the NLO corrections is done in [6], which demonstrates that the basic effect of the NLO corrections is the strong rise of the α_s argument from Q^2 to $Q_{eff}^2 = K \cdot Q^2$, where $K = 127$, i.e. $K \gg 1$.

The use of the effective argument Q_{eff}^2 in the DGLAP approach at LO approximation leads to results which are very close to the ones obtained in the case of NLO approximation: see the dot-dashed and dotted curves in Fig. 1. Thus, we hope that the effective argument represents the basic effect of the NLO corrections also in the framework of the k_T -factorization, which in some sense lies between the DGLAP and BFKL approaches.

¹Some part of higher-twist contributions was taken into account by the "freezing" procedure.

²N.Z. thanks M.V.T. Machado for useful discussion of this problem.

The results obtained in the k_T -factorization and collinear approaches based on Q_{eff}^2 argument are presented in Fig. 3. There is very good agreement between the experimental data and both theoretical approaches.

Moreover, we also present in Fig. 3 the F_L results based on the R_{world} -parameterization for the $R = \sigma_L/\sigma_T$ ratio (see [24]) (because $F_L = F_2 R/(1+R)$), and the F_2 parameterization from our previous paper [13]. The results are in good agreement with other theoretical predictions as well as with experimental data.

4 Conclusion

In the framework of k_T -factorization and perturbative QCD at LO and NLO approximations we have analyzed recent H1 preliminary data [8].

We have found very good agreement between the experimental data and collinear NLO results. The LO collinear and k_T -factorization results show a disagreement with the H1 data in the range $2 < Q^2 < 10$ GeV 2 values. It was assumed that the disagreement comes from the absence of additional higher-twist or/and NLO corrections. We shown that the account of higher-twist contributions in the form of saturation effects results in better description of the experimental data. The NLO corrections were modeled by choosing large effective scale in the QCD coupling constant. The effective corrections significantly improve also the agreement with the H1 data under consideration.

Acknowledgements

One of the authors (A.V.K.) is supported in part by Alexander von Humboldt fellowship. A.V.L. is supported in part by INTAS YSF-2002 grant № 399 and "Dinastiya" Fundation. N.P.Z. thanks the Organizing Committee for financial support and L. Jönsson for discussion of the H1 data [8].

References

- [1] W.L. van Neerven and E.B. Zijlstra, Phys. Lett. **B272**, 127 (1991); E.B. Zijlstra and W.L. van Neerven, Phys. Lett. **B273**, 476 (1991), Nucl. Phys. **B383**, 525 (1992).
- [2] D.I. Kazakov and A.V. Kotikov, Theor.Math.Phys. **73**, 1264 (1987); Nucl.Phys. **B307**, 721 (1988); E: **B345**, 299 (1990). Phys.Lett. **B291**, 171 (1992); D.I. Kazakov, A.V. Kotikov, G. Parente, O.A. Sampayo, and J. Sanchez Guillen, Phys. Rev. Lett. **65**, 1535 (1990).
- [3] A.V. Kotikov, JETP Lett. **59**, 1 (1994); Phys. Lett. **B338**, 349 (1994).
- [4] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, Phys. Lett. **B531**, 216 (2002) Eur. Phys. J. C **23**, 73 (2002); J. Pumplin, D. R. Stump, J. Huston, H.L. Lai, P. Nadolsky and W.K. Tung, JHEP **0207**, 012 (2002).
- [5] Yu.L. Dokshitzer and D.V. Shirkov, Z. Phys. **C67**, 449 (1995); W.K. Wong, Phys. Rev. **D54**, 1094 (1996).

- [6] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, *JETP. Lett.* **70**, 155 (1999).
- [7] M. Ciafaloni, D. Colferai and G.P. Salam, *Phys. Rev.* **D60**, 114036 (1999); *JHEP* **07**, 054 (2000); R.S. Thorne, *Phys. Lett.* **B474**, 372 (2000); *Phys. Rev.* **D60**, 054031 (1999); **D64**, 074005 (2001); G. Altarelli, R.D. Ball and S. Forte, *Nucl. Phys.* **B621**, 359 (2002).
- [8] E.M. Lobodzinska, *Acta Phys. Polon.* **B35**, 223 (2004); P. Newman, [hep-ex/0312018](#).
- [9] A.V. Kotikov, A.V. Lipatov and N.P. Zotov, [hep-ph/0403135](#).
- [10] S. Catani, M. Ciafaloni and F. Hautmann, *Nucl. Phys.* **B366**, 135 (1991); J.C. Collins and R.K. Ellis, *Nucl. Phys.* **B360**, 3 (1991); E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii and A.G. Shuvaev, *Sov. J. Nucl. Phys.* **53**, 657 (1991).
- [11] L.N. Lipatov, *Sov. J. Nucl. Phys.* **23**, 338 (1976); E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov. Phys. JETP* **44**, 443 (1976), **45**, 199 (1977); Ya.Ya. Balitzki and L.N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978); L.N. Lipatov, *Sov. Phys. JETP* **63**, 904 (1986).
- [12] S. Catani and F. Hautmann, *Nucl. Phys.* **B427**, 475 (1994); S. Catani, Preprint DFF 254-7-96 ([hep-ph/9608310](#)).
- [13] A.V. Kotikov, A. V. Lipatov, G. Parente and N.P. Zotov, *Eur. Phys. J. C* **26**, 51 (2002); A.V. Kotikov, A.V. Lipatov and N.P.Zotov, *Eur. Phys. J. C* **27**, 219 (2003).
- [14] J. Blumlein, *J. Phys.* **G19**, 1623 (1993).
- [15] B.Badelek, J.Kwiecinski and A. Stasto, *Z. Phys.* **C74**, 297 (1997).
- [16] Bo Andersson et al. (Small x Collaboration), *Eur. Phys. J.* **C25**, 77 (2002).
- [17] N.Nikolaev and B.M. Zakharov, *Z. Phys.* **C49**, 607 (1991); **C53**, 331 (1992).
- [18] J. Blumlein, Preprints DESY 95-121 ([hep-ph/9506403](#)).
- [19] H1 Collab.: C. Adloff et al., *Phys. Lett.* **B520**, 183 (2001).
- [20] M. Gluck, E. Reya and A. Vogt, *Z. Phys.* **C67**, 433 (1995).
- [21] J. Bartels, K. Golec-Biernat, K. Peters, *Eur. Phys. J.* **C17**, 121 (2001); E. Gotsman, E. Levin, U. Maor, L.D. McLerran, K. Tuchin, *Nucl. Phys.* **A683**, 383 (2001).
- [22] V.P. Goncalves, M.V.T. Machado, *Eur. Phys. J.* **C37**, 299 (2004).
- [23] E. Gotsman, E. Levin, M. Lublinsky and U. Maor, *Eur. Phys. J.* **C27**, 411 (2003).
- [24] SLAC Collab., L.W. Whitlow, *Phys. Lett.* **B250**, 193 (1990).